
This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.
Numerical simulation of industrial hydrocyclones performance: Role of turbulence modelling

Teja Reddy Vakamalla *, Narasimha Mangadoddy *1
*Department of Chemical Engineering, Indian Institute of Technology Hyderabad, Kandi, 502285, India.

ABSTRACT
Flow in industrial hydrocyclones is always turbulent, selection of suitable turbulence model is key for accurate predictions. This paper aims to find the appropriate turbulence model for the hydrodynamic predictions in industrial hydrocyclones. Two-phase and multiphase simulations are conducted in various size industrial hydrocyclones using volume of fluid and modified mixture models coupled with Reynolds Stress Model (RSM), Detached Eddy Simulation and Large Eddy Simulation (LES) turbulence models. Assessment of turbulence model effect on two phase flow field is made with respect to air-core, flow split, mean and turbulent velocities. The simulated flow field in 75 and 250 mm hydrocyclones is validated against literature based Laser Doppler Velocimetry, in-house high speed video and Electrical Resistance Tomography measurements. Mean density segregation contours and radial density profiles variation at different axial positions in a 350 mm dense medium cyclone is compared against Gamma Ray Tomography data. Turbulent intensity profiles are compared to display the turbulence levels. Further turbulence effect on the particles in various size hydrocyclones is analyzed by dispersion index formulation. Multiphase flow predictions in 75 and 250 mm hydrocyclones with RSM and LES models are compared against classical experimental classification performance in terms of cut size and sharpness of separation.

Keywords: Air-core, CFD, hydrocyclone, multiphase, turbulence

1 Corresponding author: narasimha@iith.ac.in; Fax: +91 40 23016032
1. Introduction

Hydrocyclones are devices used to separate or classify particles based on particle size difference in the water medium. Their usage is very wide in mineral, chemical, petroleum and bio industries due to its structural simplicity, high throughput, small volume, low cost and maintenance. Hydrocyclone mainly consists of a cylindrical section followed by a conical section attached to a single feed inlet and two product outlets named vortex finder and spigot. It works on the principle of centrifugal sedimentation. The slurry is fed tangentially to the hydrocyclone, in which larger particles are moved towards the wall and separated by centrifugal forces to the underflow. Smaller size particles follow liquid due to drag forces to the overflow. With its simple structure, the flow behavior inside the hydrocyclone is quite complex. This complexity is basically due to the presence of various size particles along with the turbulence. Due to this designers rely on empirical models for predicting the equipment performance [1, 2]. But these models can be used only within the extremities of experimental data from which the model parameters are developed.

To improve the performance of hydrocyclone, it is important to understand the fluid flow and its effect on particle separation. Therefore, experimental techniques are developed to measure the flow field in the hydrocyclones. The techniques mainly involved are Laser Doppler Velocimetry (LDV) [3-6], Particle Image Velocimetry (PIV) [6], Particle Dynamics Analyzer (PDA) [7], Positron Emission Particle Tracking (PEPT) [8] and High Speed Video imaging (HSV) [3, 9, 10]. It has helped the researchers to understand the internal flow dynamics more precisely. But most of these works are limited to two-phase or dilute flows. However, the tomography techniques like Gamma Ray Tomography (GRT) [11], Electrical Resistance Tomography (ERT) [10, 12], Electrical Impedence Tomography (EIT) [13] can be used to measure the solids concentration for dense flows. On the other hand, these experimental techniques are expensive and limited to small-scale hydrocyclone experiments. A complete research work yet to mature on high concentration slurry levels in industrial hydrocyclones. Computational fluid dynamics (CFD) is an effective technique to understand the internal flow dynamics of the hydrocyclone separator. In the past, CFD has been proved, successfully used to understand multiphase flow behavior and design modifications [14-18].
2. Literature review

The main objective of the industrial CFD simulations is to predict the performance characteristics of the equipment with reasonable accuracy in a reliable time span. Operation of hydrocyclones at high flow rates causes turbulence. Turbulence plays an important role in predicting the accurate flow field, air-core diameter and thereby particle separation efficiency. Therefore, usage of appropriate turbulence model is very important in modeling of hydrocyclones. Usage of standard \( k-\varepsilon \) and Re-Normalization Group (RNG) \( k-\varepsilon \) turbulence models [19, 20] were limited due to isotropic turbulence assumption i.e. equality of Reynolds stresses in all directions. These predictions were associated with high turbulence viscosities, unrealistic tangential velocities [21]. Delgadillo and Rajamani [19] compared the mean, turbulence flow field and air-core diameter predictions against LDV measurements in a 75 mm hydrocyclone with RNG \( k-\varepsilon \), Reynolds Stress Model (RSM) and Large Eddy Simulation (LES) turbulence models coupled with Volume of Fluid (VOF) multiphase model. LES model predictions were shown close to the experiments although it requires high computation time. Slack et al. [22] used RSM and LES to model the turbulence in 205 mm hydrocyclone. Axial and tangential velocity profiles were compared with experimental LDV measurements [5]. RSM turbulence model predictions were in good agreement with less computational cost and coarse mesh at steady state. In contrary LES captures time dependent vortex oscillations and non-equilibrium turbulence that impacts the efficiency of separation.

Brennan et al. [23] compared the tangential, axial velocity profiles predicted by RSM turbulence model with linear and quadratic pressure strain and LES model to LDV data of 75 mm hydrocyclone [4]. Observations include axial, tangential velocities with linear, quadratic pressure strains were nearly same in upper conical and cylindrical section and under predicted in the apex region. Effect of spigot and vortex finder variations were tested on the air-core diameter at different velocities. Viscosity levels were varied at a constant velocity and found less air-core diameter at high fluid viscosity. In all the cases, LES predicted accurate data compared to the experimental measurements due to improved turbulence field, thereby accurate pressure and velocity field. Narasimha’s [2] new empirical model predictions based on the results of LES indicated that particle separation is influenced by the particle size distribution and the feed solid concentration. Successful implementation of RSM model for the tangential and axial velocity...
predictions can be seen in recent literature [16, 24, 25]. Narasimha et al. [26] compared tangential and axial velocity profiles of RSM and LES models in a 100 mm Dense Medium Cyclone (DMC) [27] and concluded that axial velocity profiles were similar, but prediction of accurate tangential velocities was associated with mainly LES. Narasimha et al. [28] also compared experimental air-core position [11] in a 350 mm DMC with RSM and LES turbulence models and observed that RSM model predictions were deviated compared to the LES in the apex region. Even though RSM doesn’t model the turbulence accurately with the inherent equilibrium turbulence assumption, it can predict close velocity profiles with low computation power. Equilibrium turbulence assumes the rate of transfer of turbulence throughout length scales is constant. This is not true in the case of hydrocyclones having high swirling flows and short residence times. Contribution of energy, momentum transfer is higher through large scale eddies. Therefore solving large scale eddies and modeling small scale eddies can capture time dependent vortex oscillations and non-equilibrium turbulence [29]. From the literature it is observed that each turbulence model solves the turbulent fluctuations to some extent and models the rest. The appropriate turbulence model can be selected based on the flow physics complexity, the computation time, the required accuracy and the computational resources availability.

The presence of different size and density particles makes the multiphase phase flow modelling of hydrocyclone more challenging. The full Eulerian multiphase flow approach has been used for systems with very high particle phase concentrations, where particle/particle interactions carry a significant amount of the stress. The usage of the full Eulerian multiphase modeling approach is limited in case of hydrocyclones because of its high computational cost. Further, the implementations in commercial CFD codes have until recently been limited to using the k-ε, RSM models for turbulence. VOF [30] and Algebraic Slip Mixture (ASM) [31] are the simplified multiphase models solved for the equations of continuity, momentum of mixture. Additional equations are solved for volume fractions of additional phases. Simplified multiphase models are better than the full multiphase models in terms of uncertainties in closure relations and computation timing [31]. In the past, ASM model was successfully used to model the multiphase flows in industrial hydrocyclones and slurry flows of various designs [2, 16, 24]. The flow of solid particles can be modeled by Discrete Phase Model (DPM) [15, 32-34]. Even though DPM can show a diverse behavior of particles of different sizes and densities it cannot be used for concentrated slurries as it can only track single particle motion. DPM model doesn’t consider
the particle-particle interactions, particle volume fraction effect on the fluid medium but these interactions are important in high concentrated slurries [32].

It is believed that proper choice of turbulence model is one of the key parameter to build an accurate CFD model for hydrocyclones. Till date, the work done by various researchers are mostly limited to two phase flow field comparisons that too in a single size hydrocyclone (75 mm) [15, 23, 26, 33-35]. In most of the studies [19, 33, 34], multiphase simulations are performed mainly with DPM for dilute solids (5-10 wt%) concentration. But, industrially the hydrocyclones with larger diameter are operated at high percent of feed solids (20-50 wt%). Using the DPM based CFD model, one could mainly simulate the hydrocyclone performance predictions operating in dilute solids loading (<5 wt%) [36]. When the solids concentration exceeds 5 wt%, the presence of particles result in the generation of extra inertial stresses. The particle–fluid interaction for a single particle moving through a liquid without the presence of other particles is not valid and an increase in the slurry viscosity can also expected. The constitutive formulae, describing complex particle–fluid and particle–particle interactions are required to account extra inertial stresses, hindered settling and slurry rheology. Hence, Multiphase simulations are performed by modified ASM model with lift, drag forces and fines corrected Newtonian rheology model for slurry viscosity. Therefore, the present paper aims to find out the effect of turbulence model on the prediction of the two-phase flow field, multiphase flow field, medium segregation levels and separation size in various diameter industrial hydrocyclones (75, 250 and 350 mm) at different feed solids concentration using modified ASM model. In this paper, a hybrid turbulence model called Detached Eddy Simulation (DES), which is a combination of RSM and LES turbulence models are also tested on the ability of flow field predictions. In addition, turbulence intensity distribution in different size hydrocyclones also analyzed to understand the turbulence levels. Particle dispersion in various size hydrocyclones using two-phase flow field is analyzed critically to find out the effect of turbulence on particles.

3. Experimental data
To check the reliability of CFD model, it is essential to validate the predictions against accurate experimental measurements at various operating and design parameters. Therefore, in the present work, LDV flow field measurements of Hsieh [4] from the literature are considered to validate the two phase CFD model predictions. A separate test rig with a 75 mm conventional
hydrocyclone is also constructed to generate more experimental data like flow split, air-core diameter, cut diameter and sharpness of separation for the CFD predictions validation. As the current work is dealing with the turbulence modelling of hydrocyclone, limited experimental data with 75 mm conventional hydrocyclone is presented for the validation purposes.

3.1. 75 mm conventional test rig

The test rig with 75 mm conventional hydrocyclone used for the experimental studies is shown in Figure 1. Feed flow rate is measured by collecting the timely samples from the two outlets (underflow and overflow) for 10 seconds in two separate buckets. This process is repeated for 3 times to measure the accuracy of the measurements. After the sample collection from the feed, overflow and underflow; a small volume of sample is taken to analyze the particle size distribution. Remaining sample is kept for drying to measure the water recovery to the underflow \((R_f)\) and solids recovery to the underflow \((R_s)\). \(R_f\) and \(R_s\) values are further used in the construction of efficiency curve. The Whiten equation [37] is adopted to fit the sharpness of separation \((\alpha)\) and cut size \((d_{50})\).

Figure 1: Schematic of 75 mm hydrocyclone test rig used for the experiments.

3.2. Literature

In the present study, the experimental LDV flow filed measurements by Professor Rajamani research group, Utah University in 75 mm [4] and 250 mm [3] hydrocyclones are considered for the validation of CFD simulation studies with different turbulence models. This experimental data is extensively used for the validation of CFD predictions by many researchers over the past decade [19, 23, 26, 33-35]. These experimental test rigs (75 and 250 mm) were coupled to a LDV measurement system. Hsieh and Rajamani [4], Devulapalli [3] were able to measure mean and fluctuating tangential and axial velocity profiles at different axial positions. The error bars accounted on these LDV mean flow field measurements are summarized as the fluctuating components/RMS velocities.
4. Model Equations

4.1. Turbulence models

The CFD approach used here is same as that used by Vakamalla et al. [38]. Unsteady equations of motion for a mixture are solved by using k-ε, RNG k-ε, RSM, DES and LES turbulence models.

Momentum equation for an incompressible fluid is

\[
\frac{\partial}{\partial t} (\rho u_i) + \frac{\partial}{\partial x_j} (\rho u_i u_j) = -\frac{\partial p}{\partial x_i} + \mu \frac{\partial^2 u_i}{\partial x_j^2} + \rho g_i
\]  

(1)

Expressing the instantaneous velocity \( u_i \) into sum of averaged velocity \( \bar{u}_i \) and fluctuating velocity \( u'_i \) and time averaging the momentum equation gives

\[
\frac{\partial}{\partial t} (\rho \bar{u}_i) + \frac{\partial}{\partial x_j} (\rho \bar{u}_i \bar{u}_j) = -\frac{\partial p}{\partial x_i} + \mu \frac{\partial^2 \bar{u}_i}{\partial x_j^2} + \rho g_i - \frac{\partial}{\partial x_j} (\rho u'_i u'_j)
\]  

(2)

The term \( \rho u'_i u'_j \) is known as the Reynolds stress and it has to model to close the equation. The details of two-equation based k-ε, RNG k-ε turbulence models can be found elsewhere [36]. A brief note on the RSM, DES and LES turbulence models are given below.

4.1.1 RSM model

Unsteady transport equations given below are solved for individual Reynolds stresses \( \overline{u'_i u'_j} \) according to Gibson and Launder [39]. Quadratic pressure strain model proposed by Speziale [40] is considered for the current numerical studies.

\[
\frac{\partial}{\partial t} (\rho \overline{u'_i u'_j}) + \frac{\partial}{\partial x_k} (u_k \rho \overline{u'_i u'_j}) = \phi_j + P_j + D_{T,ij} + D_{L,ij} - \varepsilon_j
\]  

(3)

Here \( \phi_j \) is pressure strain, \( P_j \) is stress production, \( D_{T,ij} \) is turbulent diffusion, \( D_{L,ij} \) is molecular diffusion, \( \varepsilon_j \) is dissipation and modeled by the following to close the equations.

\[
\phi_j = -(C_1 \rho e + C_1^* p) b_{ij} + C_2 \rho e \left( b_{ki} b_{kj} \right) - \frac{1}{3} b_{mn} b_{mn} \delta_{ij} + \left( C_3 - C_3^* \sqrt{b_{ij} b_{ij}} \right) \rho k S_{ij}
\]
\[ C_4 \rho k \left( b_{ik} S_{jk} + b_{jk} S_{ik} - \frac{2}{3} b_{mn} S_{mn} \delta_{ij} \right) + C_5 \rho k \left( b_{ik} \Omega_{jk} + b_{jk} \Omega_{ik} \right) \] (4)

\[ p_{ij} = -\rho \left( u_i' u_j' \frac{\partial u_i}{\partial x_k} + u_j' u_i' \frac{\partial u_j}{\partial x_k} \right), \quad D_{T,ij} = \frac{\partial}{\partial x_k} \left( \frac{\mu_j \partial u_i'}{\sigma_k} \frac{\partial u_j'}{\partial x_k} \right), \quad \mu_j = C_{m} \rho \frac{k^2}{e} \]

\[ D_{L,ij} = \frac{\partial}{\partial x_k} \left( \mu_i \frac{\partial u_i'}{\partial x_k} u_j' \right), \quad \varepsilon_{ij} = \frac{2}{3} \delta_{ij} \rho e \left( 1 + \frac{2k}{a^2} \right) \] (5)

Where ‘\( b_{ij} \)’ is Reynolds stress anisotropy tensor, ‘\( \Omega_{ij} \)’ is mean vorticity tensor, ‘\( S_{ij} \)’ is mean strain rate, ‘\( \mu_i \)’ is turbulent viscosity, ‘\( k \)’ is turbulent kinetic energy, ‘\( \varepsilon \)’ is dissipation and ‘\( a \)’ is speed of sound. For the hydrocyclone flow \( \frac{2k}{a^2} \) is negligible. Constants used in the quadratic pressure strain are \( C_1 = 3.4, \ C_1^* = 1.8, \ C_2 = 4.2, \ C_3 = 0.8, \ C_3^* = 1.3, \ C_4 = 1.25, \ C_5 = 0.4, \ \sigma_k = 0.82 \) and \( C_{m} = 0.09 \).

**4.1.2 Detached Eddy Simulation (DES) model**

To reduce the computational difficulties associated with LES turbulence model; DES model is developed [41]. This model is a combination of RSM and LES turbulence models, in which RSM model is applied near the wall region where the turbulent length scale is less than maximum grid dimension and LES model is applied in the region where the turbulence length scales are greater than grid dimension. As this model is hybrid, the computation time and grid levels are expected in between RSM and LES turbulence models. In this paper, Realizable \( \kappa-\varepsilon \) model based DES model is used to run the simulation in which dissipation term is modified as

\[ \varepsilon = \frac{k^{\frac{3}{2}}}{l_{DES}}, \quad l_{DES} = \min(l_{Rke}, l_{LES}) \] (6)

\[ l_{Rke} = \frac{k^{\frac{3}{2}}}{\varepsilon}, \quad l_{LES} = C_{des} \Delta_{max} \] (7)

\( C_{des} \) is a calibration constant and has a value of 0.61 and \( \Delta_{max} \) is maximum local grid spacing.
4.1.3 LES model

Unsteady equations of motion for a slurry mixture are solved by using Large Eddy Simulation.

\[
\frac{\partial \rho_m + \partial \rho_m u_{mi}}{\partial t} = 0
\]  

(8)

\[
\frac{\partial}{\partial t} (\rho_m u_{mi}) + \frac{\partial}{\partial x_j} (\rho_m u_{mj} u_{nj}) = -\partial x_i p + \frac{\partial}{\partial x_j} (\tau_{\mu,ij} + \tau_{d,ij} + \tau_{r,ij}) + \rho_m g_i
\]  

(9)

Here \( \rho_m \) (kg/m\(^3\)) is the mixture density, \( u_{mi}, u_{mj} \) (m/s) are the \( i^{th} \) and \( j^{th} \) components of slurry velocity vector, \( x_i, x_j \) are the \( i^{th} \) and \( j^{th} \) co-ordinates, \( p_i \) (kgm/s\(^2\)) and \( g_i \) (m/s\(^2\)) is the \( i^{th} \) component of the pressure and gravity.

The viscous stress tensor in equation (9) is calculated using equation (10)

\[
\tau_{\mu,ij} = 2 \mu S_{ij}
\]  

(10)

Here \( S_{ij} \) is the mean strain rate, \( \mu \) is the effective molecular viscosity of the mixture. Sub grid scale stresses \( \tau_{r,ij} \) are solved using the Smagorinsky [42] Sub Grid Scale (SGS) model. This model proposes that the SGS eddy viscosity (\( \mu_{sgs} \)) is related to the local average grid spacing \( l_s \) and the mean strain rate \( \overline{S}_{ij} \).

\[
\tau_{r,ij} = -\mu_{sgs} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)
\]  

(11)

\[
\mu_{sgs} = \rho l_s^2 \sqrt{2 \overline{S}_{ij} \overline{S}_{ij}}
\]  

(12)

Drift tensor \( \tau_{d,ij} \), which arises from the mixture model [31] derivation accounts for the transport of momentum as a result of dispersed phase segregation.

\[
\tau_{d,ij} = \sum_{p=1}^{n} \alpha_p \rho_p u_{pm,i} u_{pm,j}
\]  

(13)

4.2. Multiphase modeling

4.2.1 VOF model

VOF model is a multiphase CFD approach used for interface tracking between the free surfaces [30] by solving the momentum equation (14). Continuity equation (15) is solved for the volume fraction of the air (\( \alpha_q \)) and this tracks the position of the air-core. In this work, geometric
reconstruction scheme with piecewise-linear approach [43] is used to track the interface between air and water. In piecewise-linear approach, the interface between air and water is assumed to have a linear slope within each cell and uses the linear shape for calculation of advection of fluids through the cell phases.

\[
\frac{\partial (\rho u_j)}{\partial t} + \frac{\partial (\rho u_j u_j)}{\partial x_j} = -\frac{\partial p}{\partial x_j} + \rho g_j + \frac{\partial}{\partial x_j} \left( \mu \left( \frac{\partial u_j}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right)
\]  

(14)

\[
\frac{\partial \alpha_q}{\partial t} + u_j \frac{\partial \alpha_q}{\partial x_i} = 0
\]

(15)

\(\alpha_q\) is volume fraction of \(q^{th}\) phase which varies between 1 and 0. \(u_j\) is \(j^{th}\) component of the velocity. The average density, viscosity are calculated by the following manner.

\[
\rho = \alpha \rho_{\text{water}} + (1 - \alpha) \rho_{\text{air}}
\]

\[
\mu = \alpha \mu_{\text{water}} + (1 - \alpha) \mu_{\text{air}}
\]

(16)

Surface tension model [44] is incorporated in Fluent\textsuperscript{TM} as a source term (equation 17) in the momentum equation. A constant value of 0.078 N/m is used for the surface tension (\(\sigma_i\)) between air and water.

\[
F_{\text{vol}} = \sigma_q \frac{\rho_k \nabla \alpha_i}{1 + \frac{1}{2} \rho_i \rho_j}
\]

(17)

4.2.2 ASM model with additional forces and rheological models

In multiphase simulations, the multiphase model is switched over from VOF to ASM model before introducing solid particles with full size distribution as dispersive phase. The slip velocity in the ASM model is disabled for the water-air interface. So, ASM model still behaves like VOF model and can able to resolve the air-core. For all other phases, the slip velocity calculations are enabled. The basic assumption behind the ASM is that dispersed phases are in equilibrium with continuous phase and such dispersed phases accelerate rapidly to terminal velocity. ASM is a simplified Eulerian multiphase CFD methodology where the equations of motion are solved for the slurry mixture and transport equations for the volume fractions of particulate phases \(p\) dispersed throughout a continuous water phase \(c\) are also solved.
Phase segregation in equation (18) is accounted for \( u_{pm,i} \) which is the drift velocity of the phase p relative to the mixture m. This is related to the slip velocity \( u_{pc,i} \), which is the velocity of the phase p relative to the continuous water phase c by the formulation

\[
u_{pm,i} = u_{pc,i} - \sum_{i=1}^{n} \frac{\alpha_i \rho_i}{\rho_m} u_{ci}
\]

\[
u_{pci} = u_{pi} - u_{c,i}
\]

\( u_{pc,i} \) is calculated algebraically in Manninen et al. [31] treatise by an equilibrium force balance and is implemented in Ansys’s Fluent\textsuperscript{TM} in a simplified form. In this work simulations are conducted with the granular options and the slip velocity calculation has been modified to include (i) a shear dependent lift force based on Saffman’s expression [45]

\[
u_{pc,i} = \frac{d_p^2(\rho_p - \rho_m)}{18 f_{rep} \mu_c} \left[ g_j - \frac{\partial}{\partial t} u_{mi} - \frac{\partial}{\partial x_j} u_{ma} + 0.75 \frac{\rho_c}{\rho_p - \rho_m} C_{lp} \varepsilon_{jk} \omega_{mj} u_{pck} \right]
\]

Here \( f_{rep} \) has been modelled with the Schiller Naumann drag law [46] but with an additional correction factor for hindered settling based on the Richardson and Zaki correlation [47]

\[
f_{rep} = \left(1 + 0.15 \text{Re}_p^{0.687}\right) \alpha_p^{-4.65}
\]

\[
\text{Re}_p = \frac{d_p \rho_p u_{pc,i}}{\mu_m}
\]

The lift force is a mechanical force, generated by solid particles as they move through a fluid, directed perpendicular to the flow direction. The inclusion of lift forces for the slip calculation will account the effect shear forces at wall on particles. The lift force expression derived by Saffman for the lift force [45] on a single particle was used here

\[
F_{lpi} = \frac{\rho_c}{8} \pi d_p^3 C_{lp} \varepsilon_{jk} \omega_{mj} u_{pck}
\]

The lift coefficient \( (C_{lp}) \) has been calculated as
\[ C_p = 4.1126 \left( \frac{\rho_c d_p^2 \omega}{\mu_c} \right) f_c \] (24)

\( f_c \) corrects the lift coefficient using the correlation proposed by Mei [48]. The equation (20) is implemented in ANSYS’s Fluent™ by a modified slip velocity \((u_{pci})\) user defined function (UDF). The slip velocity of the air phase is disabled and assumed to be zero. Here, \( \alpha_p \) is the volume fraction of particles, \( f_{rep} \) is the drag coefficient, \( d_p \) (m) is the diameter of the particle, \( \rho_c, \rho_p \) (kg/m³) are the continuous phase and particle phase density, \( \text{Re}_p \) is the Reynolds number, \( \mu_m \) (kg/ms) is the mixture/slurry viscosity, \( g_i \) (m/s²) is ‘i’ component of gravity, \( \varepsilon_{ijk} \) is the permutation tensor, \( \omega_{mij} \) is the vorticity of the mixture.

Slurry viscosity plays a vital role for the particle settling in gravity/centrifugal field at high solids concentration. Therefore, slurry viscosity is modelled with different viscosity models including both Newtonian and non-Newtonian viscosity models to study the multiphase flow behavior in the hydrocyclone. Fines corrected Newtonian model [49] is considered to modify the rheology in 75 mm, 250 mm hydrocyclones. Non-Newtonian Herschel-Bulkley model corrected with solids loading [25] is used in the predictions of 350 mm dense medium cyclones.

4.2.3 Newtonian viscosity model with feed total solids and fines correction (Nf\(_f\)\(_c\))

The effect of fine particles is high on the rheology of the mixture [50]. The increase in fines fraction can lead to a very high viscosity of suspensions. Therefore, fines correction is needed for describing the slurry behavior through Newtonian formulations. In this paper, Ishii and Mishima [51] equation is modified by including the fine fractions below 39 µm. The equation (25) was obtained by calibrating against the measured viscosity data of various mineral slurries.

\[ \frac{\mu_m}{\mu_w} = \left[ 1 - \frac{\alpha_p}{0.62} \right]^{1.55} \left( F_{39\mu} \right)^{0.39} \] (25)

4.2.4 Non-Newtonian Herschel-Bulkley model with feed total solids correction (HB)

Due to the presence of high shear rates (\( \gamma \)) within the DMC, there is a need for shear dependent viscosity model. To account this a shear dependent, the Herschel-Bulkley viscosity formulation is used as shown in equation 26. The HB model parameters i.e., yield shear stress \((\tau_0)\),
consistency (κ), flow index (n) is fitted from the experimental data of He and Laskowski [46] as shown in Figure 2. These parameters are dependent on magnetite solids volume fractions and correlated with the power law functions. This data consists of magnetite feed solids concentration varying from 5% - 30%, particle size distribution of superfine magnetite.

\[ \mu_n = \frac{\tau_0}{\gamma} + \kappa \gamma^{n-1} \]  

(26)

Figure 2: Comparing the experimental data of He and Laskowski [50] with the HB model parameters consistency (κ), flow index (n) and yield stress (τ₀) fitted.

Figure 3: Variation of (a) net mass flow rate and (b) mean tangential and axial velocities with simulation time in the 75 mm hydrocyclone.

Table 1: Dimensional details of hydrocyclones and dense medium cyclone used for the simulation studies.

5. Numerical simulation

75 mm Hsieh hydrocyclone [4], 75 mm conventional hydrocyclone [10, 38], 250 mm hydrocyclone [3], 350 mm Dutch State Mine (DSM) DMC [11] are considered for the CFD studies. Actual 350 mm DSM DMC inlet starts with cylindrical section and converges to rectangular section. To avoid complexity, mesh created here is started with rectangular section. Same volumetric flow rate is maintained as in case of circular inlet. Dimensional details of all the hydrocyclones and dense medium cyclone used in the simulation studies are provided in Table 1. All the simulations are solved with a parallel 4 node on a dual CPU Intel Xeon 2.53 GHz workstation using ANYS’s Fluent™ version 13. In the simulations, uniform velocity boundary condition is used for the feed inlet. Momentum equations are discretized using bounded central differencing scheme. PRESTO is used to solve the pressure. QUICK is used to discretize volume fraction equations. Water with a density of 998.2 kg/m³, viscosity of 0.001 kg/ms and air with a density of 1.225 kg/m³, viscosity of 1.7894x10⁻⁵ kg/ms is used to run water-air two phase simulations. In the simulations the computational domain is initialized to the properties of feed inlet. Turbulence is modeled using standard k-ε, RNG k-ε, RSM, DES and LES. At the feed inlet and pressure outlets a turbulence intensity of 10% is specified. A
restitution coefficient of 0.9 (default value in Fluent™) is used for the collisions between the particles.

Table 2: Grids and $Y^+$ values used for the simulation

Free surface between air and water (air-core) is solved using VOF model. Atmospheric pressure outlet with an air back volume fraction of 1 is used at both the outlets. This enables the simulation to generate air-core by drawing the air from both the outlets. After the air-core formation, multiphase model is changed to ASM model before introducing silica with a density of 2650 kg/m$^3$ and magnetite with a density of 4950 kg/m$^3$. The silica and magnetite particles are assumed to be spherical. A fixed time step of 1.0x10^{-4}s, 5.0x10^{-5} and 1.0x10^{-5}s are used for RSM, DES and LES turbulence model based cases. Residuals are kept in the range of 1.0x10^{-5} for continuity, velocity and turbulence parameters. Each simulation is run minimum of equipment residence time and the simulations are assumed to be converged when the residuals are less than 1.0x10^{-5}. Results displayed in this paper are plotted in XY plane. All the results are ensemble averaged over few thousand iterations equivalent to approximately 2 sec of physical time after the simulations converged. The system is considered to be converged, if the net mass flow rate between feed, overflow and underflow is oscillating around zero as shown in Figure 3 (a). The temporal variation of tangential and axial velocities after convergence is also oscillating around a constant value as shown in Figure 3 (b).

Figure 4: CFD predicted (a) mean tangential and (b) mean axial velocities by four grid sizes compared against LDV measurements at 0.12 m from top of the hydrocyclone.

6. Results and discussion

6.1. Grid independence check

Using the VOF multiphase model coupled with LES turbulence model, the flow field and air-core in a 75 mm Hsieh [4] hydrocyclone has been predicted for four different meshes 125 k, 222 k, 320 k and 950 k. Grid independence is undertaken to minimize the errors caused by underlying mesh. All the simulations are set with an inlet velocity of 2.29 m/s (1.117 kg/s) which is similar to the Hsieh’s experimental conditions. A 3D body fitted grid with 125 k nodes is used in the initial simulations. Grid independence check is performed by comparing the mean tangential and axial velocities with 4 different mesh sizes generated by grid adoption algorithm.
using $Y^+$ values and boundary nodes as shown in Figure 4. Predicted mean tangential and axial velocities of 320 k, 950 k meshes are close to each other and well compared against LDV measurements. Further, the 75 mm hydrocyclone simulations are continued with 320 k nodes and the corresponding grid is displayed in Figure 5 (a). Similarly, grid check is performed for all the hydrocyclones (75, 250 and 350 mm) used in this study. The details of nodes and corresponding $Y^+$ values for all the hydrocyclones are provided in Table 2. Grids used for the LES simulation studies are displayed in Figure 5.

Figure 5: Numerical grids of (a) 75 mm Hsieh (b) 75 mm Conventional (c) 250 mm Krebs (d) 350 mm DSM hydrocyclones used in the simulations.

6.2. Flow field predictions

Figure 6 (a), (b), (c) and (d) display the mean tangential and axial velocities predicted by VOF model coupled with different turbulence models at two different axial positions for the 75 mm Hsieh hydrocyclone. It can be noticed that standard k-$\varepsilon$, RNG k-$\varepsilon$ turbulence model predictions are completely deviated from the LDV experimental data. As noted in the literature, two equation turbulence models (standard k-$\varepsilon$, RNG k-$\varepsilon$) explored here are not suitable for the turbulence predictions in hydrocyclone [19]. RSM turbulence model based tangential flow field predictions are closely matching with the experiments towards the wall, but under predicted in the forced vortex near the air-core region. Because of its averaging procedure, RSM model may be smearing out the high velocity fluctuations near the air-core. RSM model predicted axial velocities are nearly matching with the experiments. This may be the reason for the deviation of tangential velocity prediction near the air-core with RSM turbulence model. Further, DES model which is a combination of RSM and LES turbulence model is considered for the flow field predictions. DES turbulence model predictions are also completely deviated from the experimental data. Later, LES model, which uses filtration method to separate the fluctuations, is used to predict the flow field in 75 mm Hsieh [4] hydrocyclone. By using LES, inherent vortex oscillations are directly resolved rather than modelled. It can be clearly observed, LES turbulence model is associated with accurate mean tangential and axial velocity predictions compared to remaining turbulence models at both axial positions (Figure 6 (a), (b), (c), (d)).

Root Mean Square (RMS) tangential and axial velocity predictions by all the turbulence models are also compared against the experimental data [4]. The observations include the under
prediction of RMS velocities with standard k-ε, RNG k-ε, RSM and DES turbulence models which are similar to the observations of Delgadillo [19]. Only LES model predictions are close to the experiments. The possible reason for the flow field deviations may be the inaccurate predictions of pressure field. Mean pressure contours predicted with different turbulence models are also displayed in Figure 7 (a). It can be depicted from the Figure 7 that the pressure predictions has a direct impact on the mean tangential velocity. RNG k-ε model showing low pressure drop (difference between feed pressure to over flow or underflow) hence low tangential velocity can be observed (see Figure 6). LES turbulence model predicting very high pressure drop (Figure 7 (a)) hence large velocity gradients can be seen in Figure 6 (a), (c). Whereas RSM and DES models are showing intermediate pressure gradients between RNG k-ε and LES models therefore intermediate velocities.

Table 3: CFD predicted air-core diameter and water split compared to experiments.

Figure 6: Comparison of predicted (a), (c) mean tangential, (b), (d) mean axial, (e) RMS tangential and (f) RMS axial velocities by different turbulence models at two axial positions from top of the 75 mm Hsieh [4] hydrocyclone.

Figure 7: Contours of mean (a) static pressure and (b) volume fraction of air in 75 mm Hsieh [4] cyclone.

Accurate air-core profile prediction is the ultimate aim of the two-phase flow simulations for hydrocyclones. The predicted air-core contours are shown Figure 7 (b) using various turbulence models coupled with VOF two-phase model. As depicted in Figure 7 (b), the RNG k-ε model is failed to develop the air-core. Whereas, RSM with the quadratic pressure strain model and DES with Realizable k-ε model is capable of predicting an air-core profile. However, DES model based predictions are deviated largely compared to experimental data. The non-cylindrical shape of air-core is well predicted with LES closure. The comparison between the predicted air-core diameter by RSM, DES and LES turbulence models with the experimental data shown in Figure 8. LES prediction of air-core diameter (11.5 mm) is closer to the experimental (11 mm) measurements. Local variation of air-core shape is due to the inherent nature of the LES turbulence model. Vortex finder air-core diameter above 0.05 m is much bigger than the rest of
hydrocyclone air-core due to wide size of vortex finder. The structure of the swirl flow field has a critical influence upon the inception, development and stability of the air-core.

Figure 8: Comparison of predicted air-core diameter by RSM, DES and LES turbulence models to experimental measurements in a 75mm hydrocyclone.

As discussed in the previous section, the LES model predicts the tangential velocities in liquid phase more accurately than other turbulence models. Generally the air-core free surface is an equilibrium position, at which the pressure force balances the swirl generated centrifugal acceleration force and the shear force, leading to zero radial water velocity. Since the LES model predicts accurate tangential velocity component, the air-core diameter is accurate compared to the predictions by RSM model. The smaller air-core diameter prediction by RSM turbulence model is a direct result of the under predicted tangential velocity profile. Water split and air-core diameter predictions by all the turbulence models are displayed in Table 3. From the Table 3, it can be clearly seen that the LES model predictions are closely matches with the experiments, also superior to the RSM predictions.

Further the simulations continued with RSM, DES and LES turbulence models for large diameter 250 mm hydrocyclone [3]. Corresponding mean tangential and axial velocities at two axial positions (203 and 353 mm) are displayed in Figure 9. Similar flow field observations are found with the 250 mm hydrocyclone. In which, DES turbulence model continued to under predict the mean tangential and axial velocity data. LES model is having better predictions compared to RSM turbulence model. All the turbulence model predictions are deviating near the wall. The possible reason could be experimental errors as reported earlier by Delgadillo [19].

Figure 9: Comparison of CFD predicted mean tangential and axial velocities by different turbulence models at (a) 203 mm, (b) 353 mm from top of the 250 mm hydrocyclone [3].

From the flow field comparisons in 75 mm Hsieh [4] and 250 mm hydrocyclone [3], it can be concluded that LES and RSM model predictions are superior to the lower order or hybrid turbulence models for highly swirl dominated flows. Mass flow rate, water split and air-core measured at different pressures in 75 mm conventional hydrocyclone by in house ERT and HSV are compared with the CFD predictions of RSM and LES turbulence models and displayed in Figure 10. From the Figure 10 (a), it can be depicted that mass flow rate predictions by both
RSM and LES turbulence models are very closely matching with experimental data. Water split predictions of LES model is clearly following the experimental values with an average error of 6.08%. But, RSM model predictions are deviating with the mean error of 20.39% from the experiments. Similar observations found in terms of air-core diameter predictions by RSM and LES turbulence models. RSM model over predicts the air-core diameter in the low pressure operations and reverse phenomena is observed at high pressure operations i.e. under prediction of air-core diameter. The possible reason may be the inaccurate prediction of flow field which results in the deviation of air-core diameter and water split. The mean error between RSM and LES predicted air-core with ERT is around 10.2 and 6.5%. Best predictions of air-core diameter are obtained by the LES turbulence model (9.6% average error) over RSM turbulence model (13.3%) compared to HSV imaging technique. Therefore, LES turbulence model flow field is further used to calculate the turbulent intensity and dispersion levels in various diameter hydrocyclones.

Figure 10: Comparison of RSM and LES predicted (a) water split, mass flow rate and (b) air-core diameter for 15 mm spigot against HSV and ERT in the 75 mm conventional hydrocyclone.

Figure 11: CFD predicted LZVV compared against experimental measurements in 75 and 250 mm hydrocyclone.

6.3. Locus of zero vertical velocity (LZVV)

In hydrocyclone, always two vortex flows exist. One flows towards upward (forced vortex) and another flows towards downward (free vortex). There exists a point where the vertical/axial velocity becomes zero. LZVV is defined as the locus of the fluid points at which the axial velocity becomes zero. A particle at this position has an equal chance of going either overflow or underflow. At this point, the particle is subjected to an equal outward centrifugal force and inward drag force. The drag force pulls the fine particles into the inner vortex and the centrifugal force pulls the coarse particles in to the primary/free vortex near to the wall. In general finer particles have an orbit less than LZVV and coarser particles have an orbit greater than LZVV. LZVV predictions by CFD model compared to experimental measurements in 75 mm [4] and 250 mm [3] hydrocyclones are displayed in Figure 11. Close LZVV predictions are associated with the LES turbulence model in both the hydrocyclones. LZVV shift towards the air-core is
observed with RSM turbulence model. This may be a reason for the fine cut size prediction with the RSM turbulence model (Figure 14 (a)) discussed later in section 6.6.

Figure 12: Turbulence intensity levels in (a) 75 mm, (b) 250 mm hydrocyclones and (c) comparison between the predicted turbulence intensity levels near the intersection.

6.4. Turbulence Intensity

Turbulent fluctuations inside hydrocyclones are expected to be significant due to the collision of the inlet stream with the rotating stream and flow reversal near the spigot zone. Turbulence is also directly related to large velocity gradients found inside hydrocyclones [52]. Turbulence Intensity (TI) is defined as the ratio of root mean square velocity of fluctuations ($u'$) to mean velocity ($u_{avg}$) as shown in Equation (27).

$$TI = \frac{u'}{u_{avg}}$$ (27)

The predicted turbulence intensity profiles inside two selected hydrocyclones using LES turbulence model are displayed in Figure 12. It can be observed that small size hydrocyclone (75 mm) shows very high turbulence intensities compared to bigger size hydrocyclone (250 mm). With an increase in the size of hydrocyclone, there is a reduction in the turbulence levels in the hydrocyclone.

6.5. Turbulent dispersion of particles

Dispersion index ($I_{dis}$) is defined as the ratio of dispersion energy to agglomeration energy per unit time [49]. Classification energy (centrifugal force) is taken as the agglomeration energy in case of hydrocyclones. This classification energy further can be articulated as particle diameter multiplied by classification force. Particle dispersion can be predicted by using this method [38] for hydrocyclone. Roco [53] demonstrated the importance of turbulent dispersion force in classification of particles based on particle length scale and eddy size. The calculated dispersion index (Equation 28) based on LES predictions for all the hydrocyclones are shown in Figure 13. If the $I_{dis}$ value is less than 1, then the centrifugal forces dominate the particle behavior. If the $I_{dis}$ value is greater than 10, dispersion force drives the particle behavior. Using the above criteria the dispersion levels are analyzed in the following section.
\[ I_{\text{dis}} = \left( \frac{u^*}{v_i} \right)^2 r_i \]

\[ d_p (\rho - \rho_m) \]

Where \( v_i \) is tangential velocity at a radius \( r_i \), \( u^* \) is RMS radial velocity, \( d_p \) is diameter of the particle, \( \rho_m \) and \( \rho \) is the mixture and particle density.

The dispersion index is calculated using the CFD predicted mean and turbulent flow field for 1-100 \( \mu \)m silica particles with 2650 kg/m\(^3\) density. The calculated dispersion index profiles along the radius are shown in Figure 13 (a) and dispersion index variation of the particles with hydrocyclone size is analyzed and displayed in Figure 13 (b). Fine particles (3.35 \( \mu \)m) dispersion index are above 1 (Figure 13 (a)). This states that fine particles are completely dominated by dispersion force. As the particle size increases, the dispersion index values are reduced below 1 (check 63 \( \mu \)m in Figure 13 (a)). This indicates separation is controlled by centrifugal forces for bigger size particles. From the Figure 13 (b) one can observe that the effect of turbulent dispersion is high on fine particles than the coarser particles in all the hydrocyclones. In all the hydrocyclones, particle less than 10 \( \mu \)m is significantly affected by the turbulence. With an increase in the hydrocyclone size, the particle sizes affecting due to turbulence is also increasing. A typical size of 30 \( \mu \)m is actually dominated by centrifugal forces in a 75 mm diameter hydrocyclone, whereas the same particle is affected by turbulent dispersion forces in a 350 mm dense medium cyclone.

Figure 13: Dispersion index variation of (a) different particles in a 75 mm conventional hydrocyclone (b) different particles vs hydrocyclone diameter.

6.6. Multiphase flow predictions

6.6.1. 75 mm and 250 mm hydrocyclones

Multiphase simulations are run with modified ASM model with rheology and additional forces in 75, 250 and 350 mm diameter cyclones. To demonstrate the importance of rheology and additional lift and drag forces incorporation in the ASM multiphase model, simulations are performed in 250 mm hydrocyclone [3] with DPM, standard ASM and modified ASM model with lift, drag forces and fines corrected Newtonian rheology model coupled with LES turbulence model. The efficiency curve is constructed numerically using the mass fraction of each size feed material that has reported to the underflow. The predicted separation efficiency
curves are compared against experimental measurements as shown in Figure 14 (a). It can be observed that DPM model and standard ASM model predictions are completely deviating from the experimental results at both fine and coarse end of the separation curve. In all the three models, the predicted sharpness of separation by these models is nearly close to each other. But, the predicted cut size varied significantly. The under prediction of cut size by both DPM and standard ASM models can be attributed to the inaccurate prediction of rheology, particle-fluid and particle-particle interaction forces. With increasing the feed solids percent an increase in the slurry viscosity can be expected. But, standard ASM model viscosity predictions are close to water which is unrealistic (see Figure 14 (b)). As slurry viscosity increases the particles accumulate near the apex region and surrounding particles interfere with the motion of individual particles (hindered settling). In general, the standard mixture model assumes the particle velocity is equals to its terminal velocity, which is free settling conditions for the particles. Therefore, the drag in the mixture model is also modified with additional hindered settling parameter using Richardson and Zaki correlation [47]. As observed in Figure 14 (a), the modified ASM model with rheology and additional forces is able to predict the accurate efficiency curve. An increase in the viscosity levels with the modified ASM model can also observed in the Figure 14 (b).

Figure 14: (a) Comparison of DPM, standard and modified ASM predicted and experimentally measured classification performance data [3] in a 250 mm hydrocyclone, (b) Comparison of predicted viscosities by the standard ASM and modified ASM model.

To elucidate the reason behind the cut size improvement with the modified ASM model compared to standard ASM model, both the model predicted mean tangential velocities at two axial positions (353, 648 mm from roof of the hydrocyclone) along with contours are also compared in Figure 15. It is observed that, the standard ASM model mean tangential velocity predictions are high compared to modified ASM model. The under predicted viscosity levels with low resistance for the particles may be responsible for the increased tangential velocity with the standard ASM model. Higher tangential velocities in turn results into high centrifugal forces reduce the cut size. An increase in the slurry viscosity may be a reason for the reduction of mean tangential velocities by modified ASM model and improvement in the cut size is observed (Figure 15 (a)).
Further the simulations are continued with modified ASM model in combination with RSM and LES turbulence models to simulate the particle classification. Multiphase simulations are carried out for 10 and 27.2 wt% solid concentrations in a 75 and 250 mm hydrocyclones. The effect of turbulence model on the cut size and sharpness of separation of 75 mm conventional hydrocyclone is displayed in Figure 16 (a). The predicted cut size by the RSM turbulence model is smaller when compared to experimental data. But, the predicted sharpness of separation is almost equal to the experiments. Figure 17 (a) displays the RSM and LES predicted multiphase volume fraction contours of 4.24, 10.09, 24, 48, 80.69, 114 micron size particles in a 75 mm conventional hydrocyclone. From the 4.24, 10.09 μm contours, it can be observed that more amount of fine particles are moving towards the overflow with LES turbulence model compared to RSM turbulence model. The bigger size particles greater than cut size (15 microns) are thrown towards the wall. There is a clear difference in the predictions of 24, 48 micron particle contours, in which, LES model shows particles presence even little away from the wall. Whereas, RSM model shows the 24 micron particles very closely packed towards the wall and escapes through under flow. Smaller air-core predictions (can be seen from17 (a)) by RSM model may be the reason for the over prediction of flow split (16.41%) compared to experiments (11.45%). The over prediction of flow split which in turn takes more particles to the underflow may be the reason for the finer cut size. LES model predicted flow split (11.3%) is very close to the experiments.

The predicted classification curve with 27.2% feed solids concentration is compared with the experimental data and shown in Figure 16 (b). The predicted cut size of 250 mm hydrocyclone is very close to the experimental value of ~32 microns with both the turbulence models. Also, it is noted that the imperfect classification of intermediate size particles, in particularly the 40 microns size matches experimental data quite well with both the turbulence models. RSM turbulence model is not able to predict the correct classification near the fine particles end. The reason may be RSM turbulence model inability in the prediction of mean flow field (check Figure. 6) and its association of turbulent dispersion levels for fine particles. Figure 17 (b) displays the RSM and LES predicted multiphase volume fraction contours of 4.25, 13.8, 27, 40, 55.4, 110.34 micron particles in 250 mm Krebs hydrocyclone. Figure 16 (b) classification
curve of RSM model shows a fine misplacement. From the Figure 16 (b) it can be noticed that slightly less amount fines (4.25, 13.8 microns) are moving towards the overflow in case of RSM model compared to LES turbulence model.

Figure 16: CFD predicted $d_{50}$ compared with experiments in (a) 75 mm conventional hydrocyclone for 10% solids concentration and (b) 250 mm Krebs hydrocyclone for 27.2% solids concentration.

Figure 17: CFD predicted multiphase volume fraction contours compared between RSM and LES turbulence models in (a) 75 mm Conventional cyclone for 10% solids concentration and (b) 250 mm Krebs hydrocyclone for 27.2% solids concentration.

6.6.2. 350 mm dense medium cyclone
To test the turbulence modelling effect, multiphase simulations are also conducted with RSM and LES turbulence models in a 350 mm DMC with 39.65 wt% solids concentration (Feed relative density (RD) of 1.465). Modified mixture model with non-Newtonian HB rheological [25] model is used for the multiphase simulation studies. Figure 18 (d) depicts the comparison of axial variation of GRT measured air-core radius compared to the modified CFD model predicted air-core radius. It can be observed that non-Newtonian HB model predictions based on RSM turbulence model are very close at the top portion of DMC but slightly under predicted towards the conical section. LES model predictions are slightly over predicted throughout the DMC except very few positions. Figure 18 (a) and (b) displays the RSM and LES predicted density contours of the magnetite medium segregation with modified mixture model integrated with HB rheological formulation in a 350 mm DMC. The predicted density contours are qualitatively compared to experimental GRT data [8]. It can be depicted from the Figure 18 (a) and (b) that the predicted contours of magnetite medium segregation by both the turbulence models are closely matching compared to the experimental values. RSM turbulence model based predictions are superior to LES turbulence model predictions near the air-core region. Because of small air-core and lower shear rate predictions by RSM model, more amounts of solids tried to accumulate near the air-core and better density predictions near the air-core. This is the possible reason for the over prediction of underflow volume recovery ($R_m$) with RSM turbulence model (Table 4). One can also observe a reduction in the density predictions moving towards the wall with RSM turbulence model. But with the LES turbulence model $R_m$ is reduced close to the
experimental values. Because of larger air-core, the densities near the air-core are under predicted by LES model, whereas it has improved predictions towards the wall.

Density of suspension is calculated by accounting volumetric flow rate of slurry to volumetric flow rate of water and magnetite along with respective phase volume fractions. By monitoring the mass flow rates and manually averaging the data over few hundred iterations has given the predicted density data as shown in Table 4. The CFD predicted densities are compared against experimental GRT data [11]. Qualitative assessment of contours shows reasonable medium segregation levels in the conical section with RSM turbulence model coupled with modified mixture model. But the predicted underflow, overflow densities and volume split to underflow are significantly deviated from the experimental data. LES based modified mixture model shows very close density predictions and volume split. The reason for this could be superior tangential velocities (similar to 75 mm hydrocyclone) associated with LES compared with RSM turbulence model. The radial density profiles predicted by the RSM and LES turbulence models at two axial positions 0.47m, 0.65m are shown in Figure 19 (a), (b). The radial densities predicted by RSM turbulence model are showing good improvement near the air-core. On the other side, LES turbulence model predicted densities are close to the experiments towards the wall.

Figure 18: (a) CFD predicted mean density contours of modified mixture model with RSM, (b) LES models compared with (c) experimental GRT data for feed relative density of 1.465, feed volumetric flow rate of 0.0105 m$^3$/s. (d) experimental air-core diameter compared with CFD predictions.

Figure 19: Comparison of mean radial density profiles by modified mixture model with RSM and LES turbulence models at (a) 0.47m and (b) 0.65m for a feed relative density of 1.465, feed volumetric flow rate of 0.0105 m$^3$/s.

Table 4: CFD predicted flow densities compared to experimentally measured GRT densities.

7. Conclusions

- Selection of suitable turbulence models ranging from k-ε to LES are explored for simulating accurate flow field (mean and turbulent) and thereby particle segregation in hydrocyclones.
Assessment of these turbulence models made on predicting the performance of hydrocyclones in terms of cut diameter and sharpness of separation using simulated classification curve.

Lower order (standard k-\(\varepsilon\), RNG k-\(\varepsilon\)) turbulence models are unable to predict the air-core and flow field in terms of mean and turbulent components accurately. DES model predictions are also completely under predicting the flow field.

Turbulent intensity profiles are displayed for various size hydrocyclones and observed a decay of turbulence with an increase in the hydrocyclone size.

Dispersion index formulation is used to demonstrate the effect of turbulence on the particle classification. It is observed that the critical particle size that gets influenced by the turbulence increases with the hydrocyclone size.

Multiphase flow predictions with RSM and LES turbulence models are displayed in 75 mm, 250 mm and 350 mm diameter hydrocyclones. Even though LES is computationally expensive, close predictions are associated with the LES turbulence model in all the hydrocyclones. RSM turbulence model predictions are reasonably good in bigger hydrocyclones where the turbulence levels are low. But in smaller size hydrocyclones LES predictions are superior compared to remaining turbulence models.

It is proposed that RSM model can be used in bigger size hydrocyclones due to inherent low turbulence levels. But, in smaller size hydrocyclones it is necessary to use LES turbulence model for accurate predictions.

References


[31] M. Manninen, V. Taivassalo, S. Kallio, On the mixture model for multiphase flow, in, VTT
publications, Finland, 1996.


**Figures**

Figure 1: Schematic of 75 mm hydrocyclone test rig used for the experiments.

Figure 2: Comparing the experimental data of He and Laskowski [47] with the HB model parameters consistency (κ), flow index (n) and yield stress (τ0) fitted.

Figure 3: Variation of (a) net mass flow rate and (b) mean tangential and axial velocities with simulation time in the 75 mm hydrocyclone.

Figure 4: CFD predicted (a) mean tangential and (b) mean axial velocities by four grid sizes compared against LDV measurements at 0.12 m from top of the hydrocyclone.

Figure 5: Numerical grids of (a) 75 mm Hsieh (b) 75 mm Conventional (c) 250 mm Krebs (d) 350 mm DSM hydrocyclones used in the simulations.
Figure 6: Comparison of predicted (a), (c) mean tangential, (b), (d) mean axial, (e) RMS tangential and (f) RMS axial velocities by different turbulence models at two axial positions from top of the 75 mm Hsieh [4] hydrocyclone.

Figure 7: Contours of mean (a) static pressure and (b) volume fraction of air in 75 mm Hsieh [4] cyclone.

Figure 8: Comparison of predicted air-core diameter by RSM, DES and LES turbulence models to experimental measurements in a 75mm hydrocyclone.

Figure 9: Comparison of CFD predicted mean tangential and axial velocities by different turbulence models at (a) 203 mm, (b) 353 mm from top of the 250 mm hydrocyclone [3].

Figure 10: Comparison of RSM and LES predicted (a) water split, mass flow rate and (b) air-core diameter for 15 mm spigot against HSV and ERT in the 75 mm conventional hydrocyclone.

Figure 11: CFD predicted LZVV compared against experimental measurements in 75 and 250 mm hydrocyclone.

Figure 12: Turbulence intensity levels in (a) 75 mm, (b) 250 mm hydrocyclones and (c) comparison between the predicted turbulence intensity levels near the intersection.

Figure 13: Dispersion index variation of (a) different particles in a 75 mm conventional hydrocyclone (b) different particles vs hydrocyclone diameter.

Figure 14: (a) Comparison of DPM, standard and modified ASM predicted and experimentally measured classification performance data [3] in a 250 mm hydrocyclone, (b) Comparison of predicted viscosities by the standard ASM and modified ASM model.

Figure 15: Comparison of mean tangential velocities along with contours by standard and modified ASM model at two axial positions (353, 648 mm) in a 250 mm hydrocyclone.

Figure 16: CFD predicted d50 compared with experiments in (a) 75 mm conventional hydrocyclone for 10% solids concentration and (b) 250 mm Krebs hydrocyclone for 27.2% solids concentration.
Figure 17: CFD predicted multiphase volume fraction contours compared between RSM and LES turbulence models in (a) 75 mm Conventional cyclone for 10% solids concentration and (b) 250 mm Krebs hydrocyclone for 27.2% solids concentration.

Figure 18: (a) CFD predicted mean density contours of modified mixture model with RSM, (b) LES models compared with (c) experimental GRT data for feed relative density of 1.465, feed volumetric flow rate of 0.0105 m$^3$/s, (d) experimental air-core diameter compared with CFD predictions.

Figure 19: Comparison of mean radial density profiles by modified mixture model with RSM and LES turbulence models at (a) 0.47m and (b) 0.65m for a feed relative density of 1.465, feed volumetric flow rate of 0.0105 m$^3$/s.
The graph illustrates the comparison of aircore diameter against radius for different methods: LES (solid line), DES (dashed line), RSM (dotted line), and Hsieh [7] (triangles).

- **LES** (Least Energy Simulation) shows a smooth trend with minimal oscillations.
- **DES** (Detached Eddy Simulation) displays a more oscillatory pattern compared to LES.
- **RSM** (Reynolds Stress Model) follows a trend similar to DES but with a different amplitude.
- **Hsieh [7]** (triangles) indicates data points from a specific study, showing a distinct pattern from the models.

The x-axis represents the radius in meters, ranging from 0 to 0.3, while the y-axis shows the aircore diameter in meters, ranging from 0 to 0.012.
Table 1: Dimensional details of hydrocyclones and dense medium cyclone used for the simulation studies.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet (mm)</td>
<td>25</td>
<td>45</td>
<td>75</td>
<td>65X65</td>
</tr>
<tr>
<td>Vortex finder (mm)</td>
<td>25</td>
<td>25</td>
<td>62.5</td>
<td>145</td>
</tr>
<tr>
<td>Spigot (mm)</td>
<td>12.5</td>
<td>15</td>
<td>43.75</td>
<td>105</td>
</tr>
<tr>
<td>Flow rate (kg/s)</td>
<td>1.12</td>
<td>1.52</td>
<td>9.62</td>
<td>15.38</td>
</tr>
<tr>
<td>Feed solids (wt %)</td>
<td>0</td>
<td>10</td>
<td>27.2</td>
<td>39.65</td>
</tr>
</tbody>
</table>

Table 2: Grids and $Y^+$ values used for the simulation.

<table>
<thead>
<tr>
<th>Size of hydrocyclone</th>
<th>Turbulence model</th>
<th>Grid size</th>
<th>$Y^+$ values</th>
</tr>
</thead>
<tbody>
<tr>
<td>75 mm [4]</td>
<td>RSM</td>
<td>222 k</td>
<td>30-100</td>
</tr>
<tr>
<td>75 mm [10, 36]</td>
<td>DES</td>
<td>320 k</td>
<td>10-30</td>
</tr>
<tr>
<td></td>
<td>LES</td>
<td>320 k</td>
<td>10-30</td>
</tr>
<tr>
<td>75 mm [10, 36]</td>
<td>RSM</td>
<td>273 k</td>
<td>50-100</td>
</tr>
<tr>
<td></td>
<td>LES</td>
<td>329 k</td>
<td>30-70</td>
</tr>
<tr>
<td>250 mm [3]</td>
<td>RSM</td>
<td>200 k</td>
<td>50-300</td>
</tr>
<tr>
<td></td>
<td>DES</td>
<td>321 k</td>
<td>50-100</td>
</tr>
<tr>
<td></td>
<td>LES</td>
<td>450 k</td>
<td>30-100</td>
</tr>
<tr>
<td>350 mm [11]</td>
<td>RSM</td>
<td>250 k</td>
<td>50-300</td>
</tr>
<tr>
<td></td>
<td>LES</td>
<td>600 k</td>
<td>30-100</td>
</tr>
</tbody>
</table>
Table 3: CFD predicted air-core diameter and water split compared to experiments.

<table>
<thead>
<tr>
<th>Turbulence model</th>
<th>Air-core formation</th>
<th>Air-core diameter (m)</th>
<th>Water split, Rf (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>k-ε</td>
<td>No</td>
<td>0</td>
<td>20.04</td>
</tr>
<tr>
<td>Rng k-ε</td>
<td>No</td>
<td>0</td>
<td>20.88</td>
</tr>
<tr>
<td>RSM</td>
<td>Yes</td>
<td>0.011</td>
<td>4.88</td>
</tr>
<tr>
<td>DES</td>
<td>Yes</td>
<td>0.0058</td>
<td>8.41</td>
</tr>
<tr>
<td>LES</td>
<td>Yes</td>
<td>0.0115</td>
<td>4.69</td>
</tr>
</tbody>
</table>

Table 4: CFD predicted flow densities compared to experimentally measured GRT densities.

<table>
<thead>
<tr>
<th></th>
<th>HB_RSM</th>
<th>HB_LES</th>
<th>Experimental GRT values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feed density (Kg/m$^3$)</td>
<td>1467</td>
<td>1467</td>
<td>1467</td>
</tr>
<tr>
<td>Underflow density (Kg/m$^3$)</td>
<td>1787</td>
<td>1923</td>
<td>1965</td>
</tr>
<tr>
<td>Overflow density (Kg/m$^3$)</td>
<td>1314</td>
<td>1327</td>
<td>1375</td>
</tr>
<tr>
<td>Rm (UF volume recovery)</td>
<td>0.29</td>
<td>0.16</td>
<td>0.17</td>
</tr>
</tbody>
</table>
Highlights

1. Various turbulence models are tested for accurate two-phase, multiphase flow predictions
2. Two phase flow predictions are validated against LDV, ERT and imaging techniques
3. Modified ASM with drag, lift and rheology is adopted for multiphase simulations
4. Multiphase simulations are performed for various size cyclones with RSM and LES
5. Predicted medium segregation and performance are validated against GRT and experiments